Lecturer

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Title

Introduction to Dynamic Optimization in Economics

Outline

- From Static to Dynamic Optimization: introduction to the topic; example of utility maximization from static case to dynamic case (intertemporal utility function) in discrete and continuous time.

- Examples of Dynamic Optimization problems in Economics

- Dynamic optimization problems can be seen as Optimal Control Problems or as Problems in Calculus of Variations so we provide the mathematical setting of optimal control (calculus of variations) in the discrete and in the continuous time case: state equation, pointwise constraints, set of admissible control strategies, objective functional, etc.

- Definition of Optimal Control Strategies, Optimal State Trajectories, Optimal Couples, Value Function

- Infinite horizon problems with discount: independence of the initial time.

- Feedback Control Strategies: from open-loop to closed loop controls, admissible feedback maps, optimal feedback maps.

- Translating examples of Dynamic Optimization problems (in discrete and continuous time) into problems in Optimal Control or in Calculus of Variations.

- Discussion on Optimality conditions using directional derivatives of objective functional

- Maximum Principle (PMP) in discrete time: the finite horizon case as an application of Lagrange multiplier method and connection with the Euler equation.
The infinite horizon case: brief discussion on transversality conditions in simple examples.

- Maximum Principle (PMP) in continuous time.

- Examples of applications of Maximum Principle to an AK growth model and an optimal investment model (infinite horizon case) with transversality conditions.

- Dynamic Programming (DP) Principle and Bellman equation in discrete time: first in an example and then in the general case, finite and infinite horizon.

- How to simplify the Bellman equation in the autonomous infinite horizon case.

- Optimality conditions via DP and their simplification in the autonomous infinite horizon case: the optimal feedback map.

- How to find the value function and the optimal strategies in the finite horizon case: the DP algorithm and examples.

- How to deal with the infinite horizon case: guessing the solutions in some special applied problems. Examples.

- Dynamic Programming in continuous time: value function, dynamic programming principle, dynamic programming equation, HJB equation: the finite horizon case and the infinite horizon autonomous case with discount.

- Some ideas on stochastic control/robust control/differential games

- Examples and exercises.

Exercises

- Check if a given control strategy is admissible

- Check if a given feedback map is admissible

- Given a Dynamic Optimization problem translate it as an Optimal Control (Calculus of Variations) Problem in standard form.

- Given a Dynamic Optimization problem in discrete/continuous time write the necessary conditions of the maximum principle and, in simple cases, try to solve them.

- Solve simple Dynamic Optimization problems in discrete/continuous time and in finite/infinite horizon with the Dynamic Programming method, writing the Bellman/HJB equation and solving it.

References

Notes give by the teacher: